

Spline Cúbica Natural

	t_0	t_1	t_2	t_3	t_4
$(t \text{ ou } x)$	-2	1	3	4	5
y	-1	2	6	-4	2

Gravar o sistema do slide 66 da teoria

$$h_0 = t_1 - t_0 = 1 - (-2) = 3$$

$$h_1 = t_2 - t_1 = 3 - 1 = 2$$

$$h_2 = t_3 - t_2 = 4 - 3 = 1$$

$$h_3 = t_4 - t_3 = 5 - 4 = 1$$

————— μ —————

μ_0 não existe

$$\mu_1 = 2(h_0 + h_1) = 2(3 + 2) = 10$$

$$\mu_2 = 2(h_1 + h_2) = 2(2 + 1) = 6$$

$$\mu_3 = 2(h_2 + h_3) = 2(1 + 1) = 4$$

————— μ —————

$$b_0 = \frac{1}{h_0} (y_1 - y_0) = \frac{1}{3} (2 - (-1)) = 1$$

$$b_1 = \frac{1}{h_1} (y_2 - y_1) = \frac{1}{2} (6 - 2) = 2$$

$$b_2 = \frac{1}{h_2} (y_3 - y_2) = \frac{1}{1} (-4 - 6) = -10$$

$$b_3 = \frac{1}{h_3} (y_4 - y_3) = \frac{1}{1} (2 - (-4)) = 6$$

N_0 não existe

$$N_1 = 6(b_1 - b_0) = 6(2 - 1) = 6$$

$$N_2 = 6(b_2 - b_1) = 6(-10 - 2) = -72$$

$$N_3 = 6(b_3 - b_2) = 6(6 - (-10)) = 96$$

Substituir valores no sistema

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ h_0 & u_1 & h_1 & 0 & 0 \\ 0 & h_1 & u_2 & h_2 & 0 \\ 0 & 0 & h_2 & u_3 & h_3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 0 \\ N_1 \\ N_2 \\ N_3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 3 & 10 & 2 & 0 & 0 \\ 0 & 2 & 6 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ -72 \\ 96 \\ 0 \end{bmatrix}$$

Rebo octave

$$[z_0 \ z_1 \ z_2 \ z_3 \ z_4] = [0 \ 4.2 \ -18.2 \ 28.5 \ 0]$$

A Spline cúbica natural $S(x)$
será dada por

$$S(x) = \begin{cases} S_0(x) & \text{válido de } t_0 \text{ a } t_1 \\ \frac{4.2}{18}(x+2)^3 + \left(\frac{2}{3} - \frac{1}{2}x4.2\right)(x+2) + \left(-\frac{1}{3}\right)(4-x) & \\ S_1(x) & \text{válido de } t_1 \text{ a } t_2 \\ S_2(x) & \text{válido de } t_2 \text{ a } t_3 \\ S_3(x) & \text{válido de } t_3 \text{ a } t_4 \end{cases}$$

Aproximação

Parábola: $y = a + bx + cx^2$

$$\varphi = \{1, x, x^2\}$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ \varphi_0 & \varphi_1 & \varphi_2 \end{matrix}$$

Dados:

x	0.5	0.55	0.6	0.65	0.7	0.75	0.8
y	1.2	1.0	0.7	0.4	0.1	-0.2	-0.6

Construir o sistema do slide 69

$$\underbrace{\begin{bmatrix} 1 & 0.5 & 0.2500 \\ 1 & 0.55 & 0.3025 \\ 1 & 0.60 & 0.3600 \\ 1 & 0.65 & 0.4225 \\ 1 & 0.70 & 0.4900 \\ 1 & 0.75 & 0.5625 \\ 1 & 0.80 & 0.6400 \end{bmatrix}}_A \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_x = \underbrace{\begin{bmatrix} 1.2 \\ 1.0 \\ 0.7 \\ 0.4 \\ 0.1 \\ -0.2 \\ -0.6 \end{bmatrix}}_b$$

$$x = (A^T A)^{-1} A^T b$$