Data Science in Aerospace

Elementary Probability Theory

Latest Update: *2nd March, 2026, 18:53*

ISEC Lisboa — Escola de Gestão, Engenharia e Aeronáutica

1 Departure Time

- What is the probability that, among a group of 25 airplanes scheduled to depart from an airport on the same day, at least two of them have the same departure time (to the nearest minute)?

$$1 \text{ day} = 1440 \text{ minutes}$$

$$A: \text{ Two airplanes departing at the same time}$$

$$P(A) = 1 - \frac{P(1440, 25)}{1440^{25}} \approx 0.189$$

2 Advanced Navigation System

In a fleet of 10 airplanes, 3 are equipped with advanced navigation systems.

- If two airplanes are randomly selected, what is the probability that both are equipped with advanced navigation systems?

$$A: \text{ Number of airplanes equiped with the system}$$

$$P(A = 2) = \frac{C(3, 2)}{C(10, 2)} \approx 0.07$$

3 Aircraft Dispatching

An airport has 7 identical white planes and 5 identical blue planes. The planes are dispatched randomly, one by one and without replacement, until all planes have been dispatched.

- Find the probability that the sixth plane dispatched is white and exactly 3 of the 5 planes dispatched earlier are blue.

$$P = \frac{C(5,3) \times C(7,2) \times 5}{C(12,5) \times 7} \approx 0.189$$

4 Air Traffic Communications

Suppose an airport has $n+1$ air traffic controllers, and one of them communicates a flight clearance to another, who in turn passes it on to a third, and so on successively. At each step, the controller receiving the clearance is chosen randomly among the remaining n controllers. Determine the probability that the clearance is communicated r times:

- Without being communicated back to the controller who initiated it.
- Without any controller receiving the clearance more than once.

5 Hangar Dynamics

A hangar contains aircraft weighing 5, 10, 15, and 20 tons. There are at least two aircraft of each weight. Two aircraft are randomly selected from the hangar. Let X represent the weight of the first aircraft selected and Y represent the weight of the second aircraft selected. Using the xy plane, determine:

- The sample space.
- The event $A = \{(x, y) : x = y\}$.
- The event $B = \{(x, y) : y > x\}$.
- The event $C = \{(x, y) :$ the second aircraft is twice as heavy as the first$\}$.
- The event $D = \{(x, y) :$ the first aircraft weighs 10,000 kg less than the second$\}$.
- The event $E = \{$the average weight of the two aircraft is less than 15,000 kg$\}$.

6 Airport Scheduling

Consider two aviation events A and B, where A represents the event that a plane departs on time, and B represents the event that a plane arrives on time. Suppose $P(A) + P(B) = x$ and $P(A \cap B) = y$. Determine, as a function of x and y, the probability of:

- Neither of the two events occurring.

$$\begin{aligned} P(\overline{A \cup B}) &= 1 - P(A \cup B) \\ &= 1 - (P(A) + P(B) - P(A \cap B)) \\ &= 1 - (x - y) \\ &= 1 - x + y \end{aligned}$$

- Exactly one of the two events occurring.

$$P((\overline{A} \cap B) \cup (A \cap \overline{B})) = 1 - (P(A \cap B) + P(\overline{A \cup B}))$$
$$= 1 - (y + 1 - (x - y))$$
$$= 1 - (y + 1 - x + y)$$
$$= x - 2y$$

- At least one of the two events occurring.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= x - y$$

- At most one of the two events occurring.

$$P(\overline{A} \cup \overline{B}) = P(\overline{A \cap B})$$
$$= 1 - P(A \cap B)$$
$$= 1 - y$$

7 Maintenance Oversight

Suppose two aircraft maintenance hangars, A and B, each contain 13 components. Hangar A includes 3 defective components, while hangar B contains 6 defective components. Due to an oversight, the labels identifying the hangars were lost.

- If one of the hangars is selected at random and two components are randomly removed from it, what is the probability that both components are non-defective?

C : Take two non-defective components

$$P(C) = \sum_{i=1}^{2} P(C|H_i)P(H_i) = P(A)P(C|A) + P(B)P(C|B)$$
$$= \frac{1}{2} \times \frac{C(10,2)}{C(13,2)} + \frac{1}{2} \times \frac{C(7,2)}{C(13,2)} \approx 0.423$$

- Additionally, calculate the probability that at least one of the two components is non-defective.

D : At least one component is non-defective

$$\overline{D}: \text{ Both components are defective}$$

$$P(\overline{D}) = \sum_{i=1}^{2} P(\overline{D}|H_i)P(H_i) = P(A)P(\overline{D}|A) + P(B)P(\overline{D}|B)$$
$$= \frac{1}{2} \times \frac{C(3,2)}{C(13,2)} + \frac{1}{2} \times \frac{C(6,2)}{C(13,2)} \approx 0.115$$

$$P(D) = 1 - P(\overline{D}) = 0.885$$

8 Fleet Maintenance

An airline operates a fleet of 10000 airplanes, each identified by a unique 4-digit code ranging from 0000 to 9999. During a random maintenance check, one airplane is selected at random.

- A pilot is assigned to an airplane with the code 6789. What is the probability that this airplane is selected for the maintenance check?

$$P = \frac{1}{10 \times 10 \times 10 \times 10} = 0.0001$$

- If the pilot is assigned to all airplanes whose codes have all digits identical (e.g., 1111, 2222, etc.), what is the probability that one of these airplanes is selected for the maintenance check?

$$P = \frac{10}{10 \times 10 \times 10 \times 10} = 0.001$$

- What is the probability that the airplane selected for the maintenance check has all its digits different?

$$P = \frac{P(10,4)}{10 \times 10 \times 10 \times 10} = 0.504$$

9 Boarding Gate

At an airport boarding gate, there are 4 pilots, 3 flight attendants, and 2 passengers waiting in line. What is the probability that:

- The individuals within each of these three groups (pilots, flight attendants, and passengers) are standing consecutively?

$$P = \frac{(4! \times 3! \times 2!) \times 3!}{9!} = 0.005$$

- The 2 passengers are standing together?

$$P = \frac{8 \times 2! \times 7!}{9!} \approx 0.22$$

10 Weather Forecasting

In a certain region, the weather is clear 75% of the days and stormy on the remaining 25%. A type of aviation weather forecasting system, which only predicts "Clear" or "Stormy," often makes incorrect predictions: it predicts clear weather on 10% of stormy days and predicts stormy weather on 30% of clear days.

- What is the probability that the forecasting system makes an incorrect prediction?
- What is the probability that the weather is clear on a day for which the forecast is stormy?

11 Manufacturing Reliability

Three manufacturing plants (A, B, and C) produce, respectively, 60%, 30%, and 10% of the total number of airplane parts of a certain type. For each of the plants A, B, and C, the percentage of defective airplane parts produced is 2%, 3%, and 4%, respectively.

- Assume that a container gathers all the airplane parts produced by these plants, and one part is randomly selected from this container. If the selected part is found to be defective, calculate the probability that it was produced by plant C.

$$P_i : \text{Part from container } P_i$$

$$D : \text{Defective part}$$

$$P(C|D) = \frac{P(D|C)P(C)}{P(D)} = \frac{0.04 \times 0.10}{0.6 \times 0.02 + 0.3 \times 0.03 + 0.1 \times 0.04} = 0.16$$

$$P(D) = \sum_i P(D \cap P_i) = \sum_i P(D|P_i)(P_i) = 0.02 \times 0.6 + 0.03 \times 0.3 + 0.04 \times 0.1$$

12 Structural Failure

Suppose that 42% of aviation accidents are caused by structural failures and that, for this type of accident, the probability of correctly attributing its occurrence to a structural failure is 80%. Also, assume that the probability of an accident caused by other factors being incorrectly diagnosed as due to a structural failure is 15%.

- Determine the probability that an accident, which was attributed to a structural failure as its cause, was indeed caused by such a failure.

$$F: \text{Accident due to structural failure}$$

$$D: \text{After accident, test says structural failure}$$

$$\begin{aligned} P(F|D) = \frac{P(D|F)P(F)}{P(D)} &= \\ &= \frac{0.80 \times 0.42}{P(F)P(D|F) + P(\overline{F})P(D|\overline{F}))} = \\ &= \frac{0.80 \times 0.42}{0.42 \times 0.80 + 0.58 \times 0.15} \approx 0.79 \end{aligned}$$

13 Operational Efficiency

Completing a flight operation on schedule depends on the following independent events:

$$E = \{\text{Pre-flight checks completed on time}\}$$
$$F = \{\text{Takeoff executed on time}\}$$
$$S = \{\text{Landing performed on time}\}$$

Suppose the probabilities of these events are, respectively, 0.8, 0.7, and 0.9. Calculate the probability of:

- The flight operation being completed on schedule due to the timely execution of all three activities.

$$P(E \cap F \cap S) = P(E)P(F)P(S) = 0.8 \times 0.7 \times 0.9 = 0.504$$

- The pre-flight checks being completed on time, but at least one of the other activities (takeoff or landing) not being executed on time.

$$\begin{aligned} P(E \cap (\overline{F} \cup \overline{S})) &= P(E) \times P(\overline{F} \cup \overline{S}) \\ &= P(E) \times \left(P(\overline{F}) + P(\overline{S}) - P(\overline{F} \cap \overline{S})\right) \\ &= P(E) \times \left(P(\overline{F}) + P(\overline{S}) - P(\overline{F})P(\overline{S})\right) \\ &= 0.8 \times (0.3 + 0.1 - 0.3 \times 0.1) \\ &= 0.296 \end{aligned}$$

or

$$
\begin{aligned}
P(E \cap (\overline{F} \cup \overline{S})) &= P((E \cap \overline{F}) \cup (E \cap \overline{S})) \\
&= P(E \cap \overline{F}) + P(E \cap \overline{S}) - P(E \cap \overline{F} \cap \overline{S}) \\
&= 0.8 \times 0.3 + 0.8 \times 0.1 - 0.8 \times 0.3 \times 0.1 \\
&= 0.296
\end{aligned}
$$

or

$$
\begin{aligned}
P(E \cap (\overline{F \cap S})) &= P(E) \times P(\overline{F \cap S}) \\
&= P(E) \times (1 - P(F \cap S)) \\
&= P(E) \times (1 - P(F) \times P(S)) \\
&= 0.296
\end{aligned}
$$

- The pre-flight checks being completed on time, but neither takeoff nor landing being executed on time.

$$P(E \cap \overline{F} \cap \overline{S}) = 0.8 \times 0.3 \times 0.1 = 0.024$$

14 Decision Making

A pilot is faced with a decision involving n possible actions. The pilot either knows the correct action or guesses randomly. Let p be the probability that the pilot knows the correct action. Assuming that if the pilot knows the correct action, they will choose it with probability 1 and that if the pilot guesses randomly, the probability of choosing the correct action is $1/n$.

- Verify that the probability that a pilot knew the correct action given they chose correctly is:

$$\frac{np}{1 + (n-1)p}.$$

- Calculate the probability that a randomly chosen pilot does not choose correctly, assuming $n = 5$ and $p = 0.2$.

15 Airspace Violations

Records indicate that pilots operating in a certain airspace can commit only one of two types of violations, referred to as Type I and Type II violations, with no cases observed where a pilot commits both types of violations. Among 500 pilots alerted, 100 committed a Type I violation. It is known that 10% of pilots who commit Type I violations are alerted, that 1% of all pilots commit Type I violations, and 2% of all pilots commit Type II violations.

- Calculate the probability that a pilot operating in this airspace who commits a Type II violation will be alerted.

16 Aircraft Maintenance Diagnostic Test

For a certain type of aircraft engine failure, the prevalence rate (proportion of engines with this failure in the general fleet) is 0.005. A diagnostic test for this engine failure has the following characteristics: the probability that the test results positive when applied to an engine with the failure (sensitivity of the test) is 0.99; the probability that the test results negative when applied to an engine without the failure (specificity of the test) is 0.95.

- Calculate the predictive value of the test, i.e., the probability that an engine has the failure given that the test result was positive.
- Assuming that the test was applied twice consecutively to the same engine and both times the result was positive, calculate the probability that the engine has the failure (assume that, given the state of an engine, test results in successive applications are independent). What can you conclude about the predictive value of applying the test twice consecutively?

17 Customer Satisfaction

In a market study conducted with airline passengers, it was found that 60% of the flights are operated by Airline A, while the remaining flights are operated by other airlines. The most concerning result of the study was that 25% of passengers reported dissatisfaction due to delays and poor service. It was also found that among the passengers of Airline A, 24% are dissatisfied.

- If a given passenger is dissatisfied, what is the probability that they traveled with Airline A?
- Determine the percentage of dissatisfied passengers among those who did not travel with Airline A.