Data Science in Aerospace

Random Variables

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1 Inspection

An aviation company has 7 inspectors, 2 of whom are female. For a random inspection of an aircraft, two inspectors are selected at random. Let Y denote the number of female inspectors selected.

- Define the sample space for the experiment, designating the inspectors as A, B, C, D, E (males) and F, G (females).

$$\mathcal{S} = \{\{A, B\}, \{A, C\}, \{A, D\}, \{A, E\}, \{A, E\}, \{A, F\}, ...\}$$

- Define the value of the random variable Y for each element of the sample space.

$$Y : \mathcal{S} \to \mathcal{S}' = \{0, 1, 2\}$$

$$Y(\{F, G\}) = 2$$
$$Y(\{A, F\}, \{B, F\}, \{C, F\}, ...) = 1$$
$$Y(\text{others}) = 0$$

- Define the probability and distribution functions of the random variable Y. Represent both functions in tabular form and using bar diagrams.

$$Y : \text{Number of female inspectors selected}$$

   – Probability Function

$$P(Y = 0) = \frac{C(5, 2)}{C(7, 2)} = \frac{10}{21}$$

$$P(Y = 1) = \frac{C(2, 1) \times C(5, 1)}{C(7, 2)} = \frac{10}{21}$$

$$P(Y = 2) = \frac{C(2, 2)}{C(7, 2)} = \frac{1}{21}$$

$$F(0) = \frac{10}{21}$$
$$F(1) = \frac{10}{21} + \frac{10}{21} = \frac{20}{21}$$
$$F(2) = \frac{20}{21} + \frac{1}{21} = 1$$

- Calculate the expected value and standard deviation of the random variable Y.

  - Expected value

$$E[Y] = \sum_i y P(Y = y)$$
$$= 0 \times P(Y = 0) + 1 \times P(Y = 1) + 2 \times P(Y = 2)$$
$$= 0 \times \frac{10}{21} + 1 \times \frac{10}{21} + 2 \times \frac{1}{21}$$
$$\approx 0.57$$

  - Variance

$$\text{Var}(Y) = \sum_i (y - E[Y])^2 P(Y = y)$$
$$= (0 - 0.57)^2 \frac{10}{21} + (1 - 0.57)^2 \frac{10}{21} + (2 - 0.57)^2 \frac{1}{21}$$
$$\approx 0.34$$

  - Standard deviation

$$\sqrt{\text{Var}(Y)} \approx 0.58$$

2 Aircraft Maintenance Check

An airport maintenance facility has a fleet of 10 aircraft, 3 of which are fully operational and 7 require maintenance. Three aircraft are selected at random without replacement for a safety inspection.

- What is the probability that at most one fully operational aircraft is selected?

$$P = \frac{C(7,3)}{C(10,3)} + \frac{C(7,2) \times C(3,1)}{C(10,3)} \approx 0.817$$

- If exactly one fully operational aircraft was selected in the three inspections, what is the probability that it was selected during the second inspection?

$$A : \text{Fully operational aircraft in second inspection}$$

$$B : \text{Only one operational aircraft was selected in the three inspections}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\dfrac{7 \times 3 \times 6}{P(10,3)}}{\dfrac{(7 \times 6 \times 3) + (7 \times 3 \times 6) + (3 \times 7 \times 6)}{P(10,3)}} = \frac{1}{3}$$

- Let X be the random variable representing the number of fully operational aircraft selected in the three inspections. Determine:

  - The probability mass function (PMF) of X.

$$X : \text{Number of fully operational aircraft in three inspections}$$

$$P(X=0) = \frac{C(7,3)}{C(10,3)} \approx 0.292$$

$$P(X=1) = \frac{C(3,1) \times C(7,2)}{C(10,3)} \approx 0.525$$

$$P(X=2) = \frac{C(3,2) \times C(7,1)}{C(10,3)} \approx 0.175$$

$$P(X=3) = \frac{C(3,3)}{C(10,3)} \approx 0.008$$

  - The cumulative distribution function (CDF) of X.

$$F(0) = 0.292$$
$$F(1) = 0.292 + 0.525 \approx 0.817$$
$$F(2) = 0.817 + 0.175 \approx 0.992$$
$$F(3) = 0.992 + 0.008 = 1$$

  - The expected value and variance of X.
    * Expected value

$$E[X] = \sum_x x P(X=x)$$
$$= 0 \times P(X=0) + 1 \times P(X=1) + 2 \times P(X=2) + 3 \times P(X=3)$$
$$\approx 0.899$$

    * Variance

$$\text{Var}(X) = \sum_x (x - E[X])^2 P(X = x)$$
$$= (0 - E[X])^2 P(X = 0) + (1 - E[X])^2 P(X = 1)+$$
$$+(2 - E[X])^2 P(X = 2) + (3 - E[X])^2 P(X = 3)$$
$$\approx 0.489$$

- Answer first three points again, assuming that the three inspections were conducted with replacement.

To be solved

3 Engine Testing

At an airline maintenance facility, there are three identical aircraft engines of the same model that operate independently. The probability of each engine failing during a given time period is 0.1. Let X be the random variable representing the number of engines that are still operational at the end of this time period. Determine:

- The probability mass function of X.

$$P(X = 0) = 1 \times 0.1 \times 0.1 \times 0.1 = 0.001$$
$$P(X = 1) = 3 \times 0.9 \times 0.1 \times 0.1 = 0.027$$
$$P(X = 2) = 3 \times 0.9 \times 0.9 \times 0.1 = 0.243$$
$$P(X = 3) = 1 \times 0.1 \times 0.9 \times 0.9 = 0.729$$

- The cumulative distribution function of X.

$$F(0) = 0.001$$
$$F(1) = 0.028$$
$$F(2) = 0.271$$
$$F(3) = 1.000$$

- The expected value, mode, median, and variance of X.

$$\text{Mode} = 3$$

$$\text{Median} = 2 + \frac{3 - 2}{1 - 0.271}(0.5 - 0.271) \approx 2.31$$

$$E[X] = \sum_i x P(X = x)$$

$$\text{Var}(X) = \sum_x (x - E[X])^2 P(X = x)$$

4   Airline Booking Service

At an airline ticket counter, the time interval Δt (in minutes) between any two consecutive flight reservation calls is modeled by the following probability density function:

$$f(\Delta t) = \begin{cases} e^{-\Delta t}, & \text{if } \Delta t \geq 0, \\ 0, & \text{if } \Delta t < 0. \end{cases}$$

- Calculate $P(\Delta t > 2)$.

$$\begin{aligned} P(\Delta t > 2) &= \int_2^{+\infty} e^{-\Delta t} \, d\Delta t \\ &= \lim_{k \to +\infty} \int_2^k e^{-\Delta t} \, d\Delta t \\ &= \lim_{k \to +\infty} \left[-e^{-\Delta t} \right]_2^k \\ &= \lim_{k \to +\infty} \left(-e^{-k} + e^{-2} \right) \\ &= e^{-2} \end{aligned}$$

- Calculate $P(\Delta t > 3)$.

$$\begin{aligned} P(\Delta t > 2) &= \int_3^{+\infty} e^{-\Delta t} \, d\Delta t \\ &= e^{-3} \end{aligned}$$

- Calculate the conditional probability $P(\Delta t > 3 \mid \Delta t > 1)$.

$$\begin{aligned} P(\Delta t > 3 \mid \Delta t > 1) &= \frac{P(\Delta t > 3 \wedge \Delta t > 1)}{P(\Delta t > 1)} \\ &= \frac{\int_3^{+\infty} e^{-\Delta t} \, d\Delta t}{\int_1^{+\infty} e^{-\Delta t} \, d\Delta t} \\ &= \frac{e^{-3}}{e^{-1}} = e^{-2} \end{aligned}$$

5   Flight Altitude

Let X represent the altitude (in kilometers) of an aircraft during a specific flight phase. The probability density function of X is given by:

$$f(x) = \begin{cases} k(9x - 6x^2 + x^3), & \text{if } 0 \leq x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

- Determine the value of k to ensure $f(x)$ is a valid probability density function. Plot the function $f(x)$.
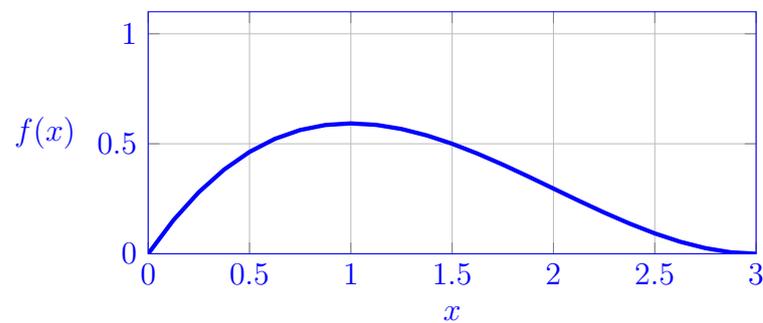
$$\int_{-\infty}^{+\infty} f(x)\, dx = 1$$

$$\int_0^3 k(9x - 6x^2 + x^3)\, dx = 1$$

$$\left[k \left(\frac{9x^2}{2} - \frac{6x^3}{3} + \frac{x^4}{4} \right) \right]_0^3 = 1$$

$$k = 1 \bigg/ \left[\frac{9x^2}{2} - \frac{6x^3}{3} + \frac{x^4}{4} \right]_0^3$$

$$k = 4/27$$

- Calculate the probabilities $P(X \le 1.5)$, $P(X \ge 2.0)$ and $P(1.0 \le X \le 2.5)$.

$$P(X \le 1.5) = \int_{-\infty}^{1.5} \frac{4}{27}(9x - 6x^2 + x^3)\, dx$$

$$= \int_0^{1.5} \frac{4}{27}(9x - 6x^2 + x^3)\, dx$$

$$\approx 0.688$$

$$P(X \ge 2.0) = \int_2^{+\infty} \frac{4}{27}(9x - 6x^2 + x^3)\, dx$$

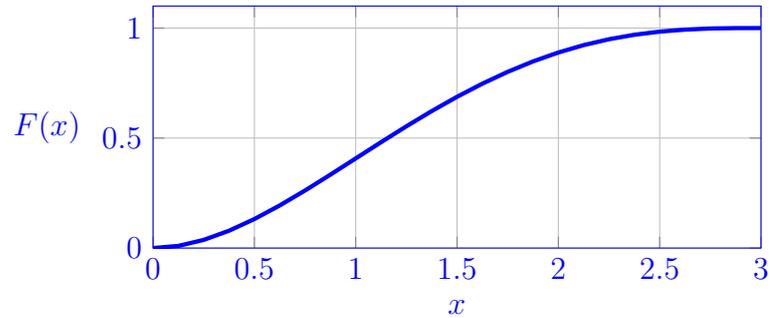$$= \int_2^3 \frac{4}{27}(9x - 6x^2 + x^3)\, dx$$

$$\approx 0.111$$

$$P(1.0 \le X \le 2.5) = \int_{1.0}^{2.5} \frac{4}{27}(9x - 6x^2 + x^3)\, dx$$

$$\approx 0.576$$

- Compute the cumulative distribution function $F(x)$, and plot it graphically.

$$F(x) = P(X \le x) = \int_0^x \frac{4}{27}(9x - 6x^2 + x^3)\,dx$$

$$= \frac{4}{27}\left(\frac{9x^2}{2} - \frac{6x^3}{3} + \frac{x^4}{4}\right)$$

6   Training Sessions

Consider a discrete random variable X, representing the number of successful landings by a pilot during a training session, with the following probability function:

$$P(X = x) = \begin{cases} ax & \text{if } x \in \{1, 2, 3\}, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a real constant.

- Determine the value of a.

$$\sum_x P(X = x) = 1$$
$$P(X = 1) + P(X = 2) + P(X = 3) = 1$$
$$a + 2a + 3a = 1$$
$$a = 1/6$$

- Determine the cumulative distribution function (CDF) of X.

$$F(x) = \sum_{i=1}^x P(X = x)$$
$$F(1) = P(X = 1) = 1/6$$
$$F(2) = P(X = 1) + P(X = 2) = 1/2$$
$$F(3) = P(X = 1) + P(X = 2) + P(X = 3) = 1$$

- Calculate the mode, median, and expected value of X.

- Mode

$$X = 3$$

- Median

$$X = 2$$

- Expected Value

$$
\begin{aligned}
E[X] &= \sum_i xP(X = x) \\
&= 1 \times P(X = 1) + 2 \times P(X = 2) + 3 \times P(X = 3) \\
&= 1 \times 1/6 + 2 \times 1/3 + 3 \times 1/2 \\
&\approx 2.333
\end{aligned}
$$

7 Fuel Efficiency

An aircraft's fuel efficiency (in miles per gallon) can be considered a random variable X with the probability density function given by:

$$
f(x) = \begin{cases} \dfrac{3}{5}10^{-5}x(100 - x), & \text{if } 0 \leq x \leq 100, \\ 0, & \text{otherwise.} \end{cases}
$$

Suppose the profit L obtained from operating the aircraft (per flight hour) depends on the fuel efficiency according to the relationship:

$$L = C_1 + C_2 X,$$

where C_1 and C_2 are constants.

- Calculate the expected value and variance of the profit per flight hour.

    - Expected Value

$$
\begin{aligned}
E[X] &= \int_{-\infty}^{+\infty} x f(x) \, dx \\
&= \int_0^{100} x \left(\frac{3}{5}10^{-5}x(100 - x) \right) \, dx \\
&= \frac{3}{5}10^{-5} \int_0^{100} 100x^2 - x^3 \, dx \\
&= 50
\end{aligned}
$$

$$
\begin{aligned}
E[L] &= E[C_1 + C_2 X] \\
&= E[C_1] + E[C_2 X] \\
&= C_1 + C_2 E[X] \\
&= C_1 + 50C_2
\end{aligned}
$$

– Variance

$$\begin{aligned}
\mathsf{Var}(X) &= \int_{-\infty}^{+\infty} (x - E[X])^2 f(x) \, dx \\
&= \int_{0}^{100} (x - 50)^2 \left(\frac{3}{5} 10^{-5} x(100 - x) \right) \, dx \\
&= 500
\end{aligned}$$

$$\begin{aligned}
\mathsf{Var}(L) &= \mathsf{Var}(C_1 + C_2 X) \\
&= \mathsf{Var}(C_2 X) \\
&= C_2^2 \mathsf{Var}(X) \\
&= 500 C_2^2
\end{aligned}$$

8 Maintenance Process

Assume that, in a certain aviation maintenance process, the temperature of the engine oil recorded at the start of each shift in a specific aircraft follows a distribution with an expected value of $153°$F and a standard deviation of $7°$F.

- Calculate these two parameters of the temperature distribution when expressed in Celsius.

$$C = \frac{5}{9}(F - 32)$$

  – Expected Value

$$\begin{aligned}
E[C] &= E\left[\frac{5}{9}(F - 32) \right] \\
&= E\left[\frac{5}{9} F \right] + E\left[-32 \times \frac{5}{9} \right] \\
&= \frac{5}{9} E[F] - \frac{160}{9} \\
&\approx 67.2 \,°\mathsf{C}
\end{aligned}$$

  – Variance

$$\begin{aligned}
\mathsf{Var}(C) &= \mathsf{Var}\left(\frac{5}{9}(F - 32) \right) \\
&= \mathsf{Var}\left(\frac{5}{9} F \right) \\
&= \frac{25}{81} \mathsf{Var}(F)
\end{aligned}$$

  – Standard Deviation

$$\begin{aligned}
\sqrt{\mathsf{Var}(C)} &= \sqrt{ \frac{25}{81} \mathsf{Var}(F) } \\
&\approx 3.89 \,°\mathsf{C}
\end{aligned}$$