# Computational Applied Mathematics

***Examples***

Latest Update: *15th November, 2026, 21:37*

ESCOLA
DE GESTÃO,
ENGENHARIA
E AERONÁUTICA

## Taylor Series

1. An airplane changes its altitude over time as

$$z(t) = 100 + 10\sin(2\pi f t),$$

where $f = 0.05\,\text{Hz}$. Approximate $z$ at $t = 4.5$ with a Taylor series and estimate through it the altitude when $t = 5$. Determine the absolute and relative error of such estimate.

2. During a flight, the aerodynamic drag acting on an aircraft varies with its airspeed according to

$$D(V) = 0.04V^2 + 2V,$$

where $D$ is the drag force in newtons and $V$ is the airspeed in $\text{m}\,\text{s}^{-1}$.

Using a Taylor series expansion around $V = 80\,\text{m}\,\text{s}^{-1}$, approximate the drag force when the aircraft reaches $V = 85\,\text{m}\,\text{s}^{-1}$. Use the Taylor polynomial up to the third-order term.

3. During a landing approach, the distance covered by an aircraft after deploying its airbrakes is approximated by the following function:

$$s(t) = 80t - 3t^2 + 0.2t^3,$$

where $s$ is the distance travelled in meters and $t$ is the time elapsed in seconds.

Using a Taylor series expansion around $t = 4\,\text{s}$, approximate the distance travelled at $t = 4.5\,\text{s}$. Use the Taylor polynomial up to the third-order term.

# Numerical Derivation

1. An airplane changes its altitude over time as

$$z(t) = 100 + 10\sin(2\pi f t),$$

where $f = 0.05\,\text{Hz}$. Estimate the vertical velocity, $\dot{z}$, at $t = 4.5$ and vertical acceleration, $\ddot{z}$, at $t = 5.2$.

2. During a flight test, the airspeed of an aircraft changes over time according to

$$V(t) = 70 + 15\sin(0.2t),$$

where $V$ is the airspeed in $\text{m}\,\text{s}^{-1}$ and $t$ is the time in seconds.

Using numerical differentiation with $h = 0.01$, estimate the aircraft acceleration, $\dot{V}$, at $t = 3.0$ and the rate of change of acceleration (jerk), $\ddot{V}$, at $t = 4.0$.

3. During an aircraft landing, the aerodynamic drag force varies with the aircraft's velocity according to

$$D(V) = 0.02V^2 + 5\sqrt{V},$$

where $D$ is the drag force in newtons and $V$ is the aircraft velocity in $\text{m}\,\text{s}^{-1}$.

Using numerical differentiation with $h = 0.1$, estimate the rate of change of drag force with respect to velocity, $\dfrac{dD}{dV}$, at $V = 60\,\text{m}\,\text{s}^{-1}$ and the second derivative, $\dfrac{d^2D}{dV^2}$, at $V = 80\,\text{m}\,\text{s}^{-1}$.

# Zeros of Functions

**Solve the following exercises using the bisection, *Regula Falsi*, Newton's and Secant Methods.**

1. Determine the velocity $V$ at which the thrust equals the drag, that is, $D(V) = T(V)$.

   - **Drag function:** $D(V) = 0.01V^2 + 10\sqrt{V}$
   - **Thrust function:** $T(V) = 3000 - 10V$

2. During the operation of a wind turbine, the aerodynamic power extracted from the wind depends on the wind speed, while the power losses associated with the electrical generator increase with rotational speed. For a simplified operating model, these two effects are represented by the following functions:

   $$P_A(V) = 0.015V^2 + 8\sqrt{V}$$

   and

   $$P_L(V) = 2500 - 8V,$$

   where $P_A$ and $P_L$ are expressed in watts and $V$ represents the wind speed in $\mathrm{m\,s^{-1}}$.

   Determine the wind speed at which the available aerodynamic power equals the power losses, that is, $P_A(V) = P_L(V)$.

3. During the landing phase, an aircraft uses a combination of wheel brakes, aerodynamic spoilers and thrust reversers to decelerate. The forces generated by these systems are modelled as functions of the aircraft velocity:

   The wheel braking force is given by

   $$F_B(V) = 1800 + 4V,$$

   the aerodynamic drag by

   $$D(V) = 0.025V^2 + 6\sqrt{V},$$

   the additional drag generated by the spoilers by

   $$F_S(V) = 0.02V^2,$$

   and the reverse thrust by

   $$F_R(V) = 1200 + 2V.$$

   All forces are expressed in newtons and $V$ is the aircraft velocity in $\mathrm{m\,s^{-1}}$.

   Determine the aircraft velocity at which the total decelerating force reaches 3500 N by solving the corresponding nonlinear equation.

# Maxima and Minima of Functions

**Solve the following exercises using the Golden Section, Quadratic Interpolation and Newton's Methods.**

1. Determine the lift and drag coefficients that maximize the aerodynamic efficiency defined as $C_L/C_D$. Consider that
$$C_D = 0.02 + 0.05C_L^2.$$

2. An aircraft engine operates at different rotational speeds. The fuel consumption rate as a function of engine rotational speed is approximated by
$$F(N) = 0.0002N^2 - 0.12N + 35,$$
where $F$ is the fuel consumption rate in $\mathrm{kg\,h^{-1}}$ and $N$ is the engine rotational speed in rotations per minute. Determine the engine rotational speed, $N$, that minimizes the fuel consumption rate and calculate the corresponding minimum fuel consumption.

3. During the preliminary design of an aircraft wing, the structural mass is estimated as a function of the wing thickness, $t$, according to
$$m(t) = 120 + 80t + \frac{200}{t},$$
where $m$ is the wing mass in kilograms and $t$ is the normalized wing thickness (dimensionless). Determine the wing thickness that minimizes the structural mass and calculate the corresponding minimum mass.

# Linear Systems of Equations

**Solve the following exercises using the Gauss Elimination, Gauss-Jordan Elimination, Jacobi and Gauss-Seidel Methods.**

1. For an aircraft structure, the equilibrium of forces in three orthogonal directions is governed by the following systems of equations. The unknowns $F_1$, $F_2$, $F_3$ are the tensions (in kN) in three structural members. Determine the tension in each member.

$$\begin{cases} 3F_1 + 4F_2 + 3F_3 = 10 \\ F_1 + 5F_2 - F_3 = 7 \\ 6F_1 + 3F_2 + 7F_3 = 15 \end{cases}$$

2. During the operation of an aircraft hydraulic system, three hydraulic actuators generate forces that must satisfy equilibrium conditions along three orthogonal directions. The unknowns $F_1$, $F_2$ and $F_3$ represent the forces (in kN) generated by each actuator. The equilibrium equations are given by

$$\begin{cases} 4F_1 + 2F_2 + F_3 = 14 \\ 2F_1 + 5F_2 - F_3 = 13 \\ F_1 - F_2 + 4F_3 = 8 \end{cases}.$$

   Determine the force generated by each actuator.

3. During the design of an aircraft landing gear, three structural components transmit loads between the landing gear assembly and the aircraft fuselage. The unknowns $R_1$, $R_2$ and $R_3$ represent the reaction forces (in kN) at three structural connections. The equilibrium conditions are described by

$$\begin{cases} 5R_1 + 2R_2 + R_3 = 18 \\ 2R_1 - R_2 + 5R_3 = 15 \\ R_1 + 4R_2 + 2R_3 = 16 \end{cases}.$$

   Determine the reaction force at each structural connection.

# Nonlinear Systems of Equations

1. Each time a GPS is used, a system of nonlinear equations of the form

$$\begin{aligned}(x-a_1)^2+(y-b_1)^2+(z-c_1)^2&=[C(t_1-D)]^2\\(x-a_2)^2+(y-b_2)^2+(z-c_2)^2&=[C(t_2-D)]^2\\(x-a_3)^2+(y-b_3)^2+(z-c_3)^2&=[C(t_3-D)]^2\\(x-a_4)^2+(y-b_4)^2+(z-c_4)^2&=[C(t_4-D)]^2\end{aligned}$$

   is solved for the $(x, y, z)$ coordinates of the receiver. For each satellite $i$, the locations are $(a_i, b_i, c_i)$, and $t_i$ is the synchronized transmission time from the satellite. Further, $C$ is the speed of light, and $D$ is the difference between the synchronized time of the satellite clocks and the earth-bound receiver clock.

   Given the satellite coordinates $(15600, 7540, 20140)$, $(18760, 2750, 18610)$, $(17610, 14630, 13480)$, $(19170, 610, 18390)$, the time intervals $(0.07074, 0.07220, 0.07690, 0.07242)$ and using the initial guess $(0, 0, 6370, 0)$, use the Newton's method to approximate the position of the receiver of the GPS signals.

2. An aircraft is flying in steady, level flight. The aerodynamic lift and drag coefficients depend on the angle of attack $\alpha$ according to

$$C_L = 0.2 + 5.5\alpha, \qquad C_D = 0.025 + 0.08C_L^2.$$

   The airspeed $V$ and angle of attack $\alpha$ can be determined by solving the following nonlinear system:

$$\frac{1}{2}\rho V^2 S C_L(\alpha) - W = 0 \qquad \text{and} \qquad \frac{1}{2}\rho V^2 S C_D(\alpha) - T = 0$$

   where $\rho$ is the air density, $S$ is the wing area, $W$ is the aircraft weight, and $T$ is the available engine thrust.

   Given the parameters $\rho = 1.225$ kg/m$^3$, $S = 16$ m$^2$, $W = 12000$ N and $T = 2000$ N, use Newton's method to determine the airspeed and angle of attack of the aircraft. Consider the initial guess $(V_0, \alpha_0) = (50, 0.05)$.

3. During a manoeuvre, an aircraft performs a coordinated turn at a constant altitude. The relationship between the bank angle $\phi$ and the turn radius $R$ is governed by the centripetal acceleration and the vertical force equilibrium. The unknown bank angle $\phi$ and airspeed $V$ must satisfy the following nonlinear system:

$$\frac{V^2}{R} - g\tan(\phi) = 0 \qquad \text{and} \qquad \frac{1}{2}\rho V^2 S C_L(\phi) - \frac{W}{\cos(\phi)} = 0$$

   where $g$ is the gravitational acceleration, $\rho$ is the air density, $S$ is the wing area, $W$ is the aircraft weight, and the lift coefficient is approximated by

$$C_L(\phi) = 0.3 + 4.5\phi.$$

   Given the parameters $R = 500$ m, $g = 9.81$ m/s$^2$, $\rho = 1.225$ kg/m$^3$, $S = 16$ m$^2$ and $W = 12000$ N, use Newton's method to determine the airspeed and bank angle of the aircraft. Consider the initial guess $(V_0, \phi_0) = (60, 0.2)$.