ISEC Lisboa

Licenciatura em Ciências Aeronáuticas e do Espaço

Dinâmica de Fluidos Computacional

ESCOLA DE GESTÃO, ENGENHARIA E AERONÁUTICA

1. Water flows steadily through a T-junction, as illustrated in Figure 1. The flow enters the control volume through section 1 and exits through sections 2 and 3. Assume uniform velocity profiles across all three sections.

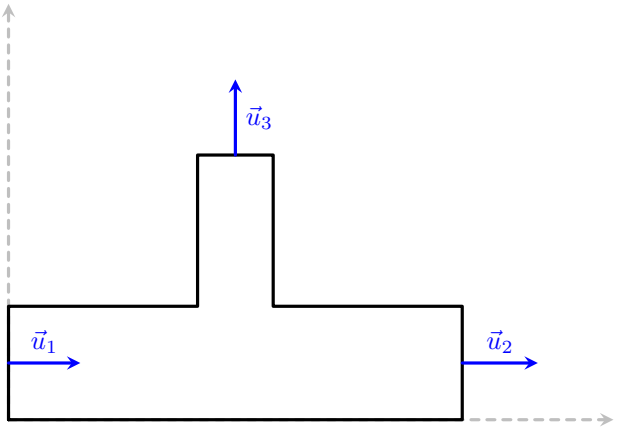


Figure 1: T-junction control volume.

The following operating conditions are given:

- Fluid density: $\rho = 1000$ kg/m^3.
- Inlet velocity: $u_1 = 5$ m/s.
- Outlet velocity at section 2: $u_2 = 4$ m/s.
- Outlet velocity at section 3: $u_3 = 6.67$ m/s.
- Cross-sectional area at section 1: $A_1 = 0.04$ m^2.
- Cross-sectional area at section 2: $A_2 = 0.025$ m^2.
- Cross-sectional area at section 3: $A_3 = 0.015$ m^2.

Assume steady-state, incompressible flow, atmospheric pressure at all three sections, and neglect gravitational and viscous effects.

(a) Verify that the given flow conditions satisfy the conservation of mass equation.

(b) Starting from the integral conservation of linear momentum equation, determine the net external force acting on the fluid inside the control volume, expressed in Cartesian coordinates.

2. Consider the steady flow of a fluid through an inclined T-junction, as illustrated in Figure 2. The flow enters the control volume through section 1 and exits through sections 2 and 3. Assume uniform velocity profiles across all three sections.

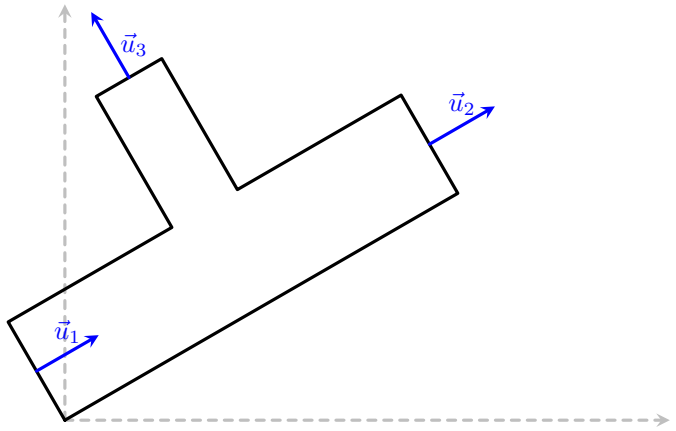


Figure 2: T-junction control volume.

The following operating conditions are given:

- Section 1:
  - Density: $\rho_1 = 1000$ kg/m^3.
  - Cross-sectional area: $A_1 = 0.040$ m^2.
  - Velocity magnitude: $u_1 = 5.0$ m/s.
  - Flow direction: $\theta_1 = 30°$.
- Section 2:
  - Density: $\rho_2 = 900$ kg/m^3.
  - Cross-sectional area: $A_2 = 0.025$ m^2.
  - Velocity magnitude: $u_2 =$?.
  - Flow direction: $\theta_2 = 30°$.
- Section 3:
  - Density: $\rho_3 = 800$ kg/m^3.
  - Cross-sectional area: $A_3 = 0.015$ m^2.
  - Velocity magnitude: $u_3 = 7.0$ m/s.
  - Flow direction: $\theta_3 = 120°$.

Assume that the pressure at all three sections is atmospheric, and neglect gravitational and viscous effects.

(a) Starting from the integral conservation of mass equation, determine the unknown velocity magnitude u_2.

(b) Starting from the integral conservation of linear momentum equation, determine the net external force acting on the fluid inside the control volume. Express the result in Cartesian coordinates.

3. Consider the three-dimensional control volume illustrated in the figure.

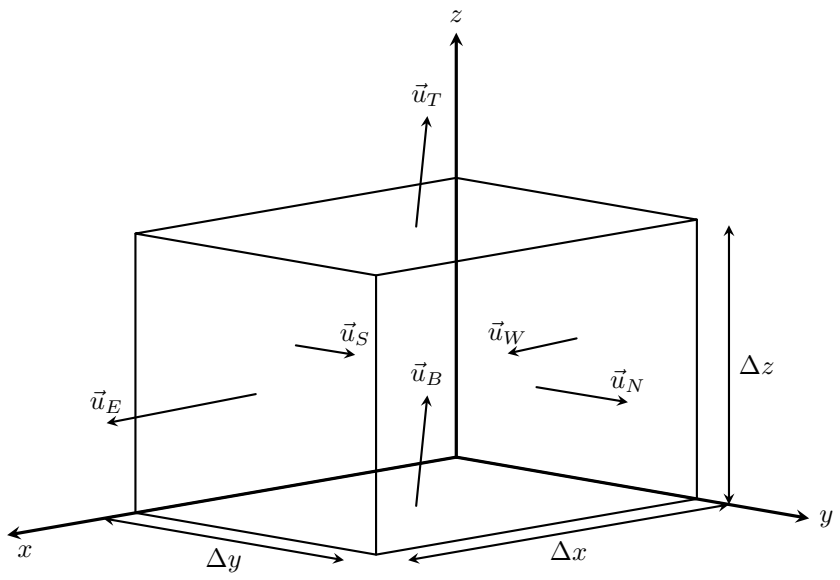


Figure 3: TEST

The dimensions of the control volume are

$$\Delta x = 2.0 \text{ m}, \qquad \Delta y = 1.5 \text{ m}, \qquad \Delta z = 1.25 \text{ m}.$$

The velocity vectors at the six faces of the control volume are given by

$$\begin{aligned}
\vec{u}_S &= (0.30,\ 1.20,\ -0.15) \text{ m/s}, \\
\vec{u}_N &= (-0.20,\ 0.80,\ 0.25) \text{ m/s}, \\
\vec{u}_W &= (1.50,\ -0.30,\ 0.20) \text{ m/s}, \\
\vec{u}_E &= (-0.40,\ 0.25,\ -0.10) \text{ m/s}, \\
\vec{u}_B &= (0.15,\ -0.20,\ 0.60) \text{ m/s}, \\
\vec{u}_T &= (u_T,\ v_T,\ w_T) \text{ m/s}.
\end{aligned}$$

The density and its time derivative are uniform throughout the control volume:

$$\rho = 1.2 \text{ kg/m}^3, \qquad \frac{\partial \rho}{\partial t} = -0.2 \text{ kg/(m}^3 \text{ s)}.$$

Using the integral conservation of mass equation, determine the unknown velocity component w_T.

$$\iiint_V \nabla \cdot \mathbf{F} \, \mathrm{d}V = \oiint_S \mathbf{F} \cdot \hat{\mathbf{n}} \, \mathrm{d}S$$

4. Consider the three-dimensional control volume illustrated in the figure.

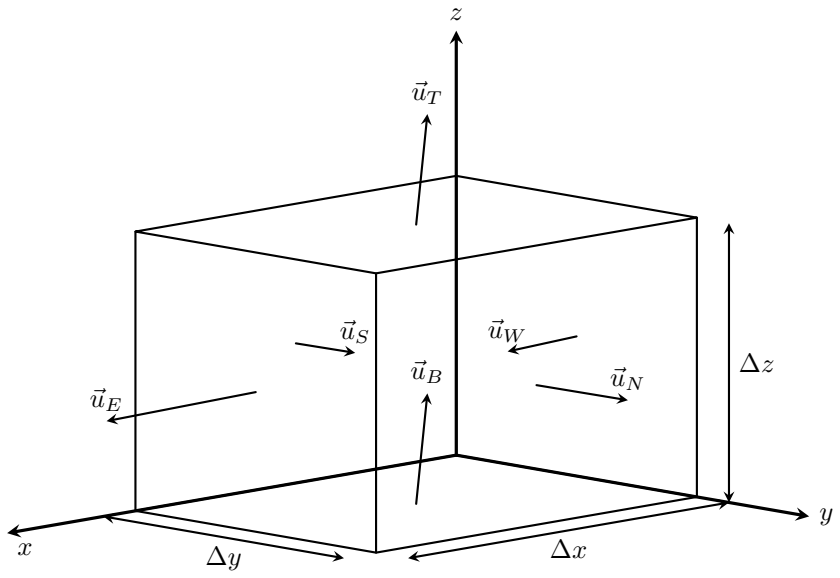


Figure 4: TEST

The dimensions of the control volume are

$$\Delta x = 2.0 \text{ m}, \qquad \Delta y = 1.5 \text{ m}, \qquad \Delta z = 1.25 \text{ m}.$$

The velocity vectors and densities at the six faces of the control volume are given by

$$\begin{aligned}
\vec{u}_S &= (0.30,\ 1.20,\ -0.15) \text{ m/s}, \\
\vec{u}_N &= (-0.20,\ 0.80,\ 0.25) \text{ m/s}, \\
\vec{u}_W &= (1.50,\ -0.30,\ 0.20) \text{ m/s}, \\
\vec{u}_E &= (-0.40,\ 0.25,\ -0.10) \text{ m/s}, \\
\vec{u}_B &= (0.15,\ -0.20,\ 0.60) \text{ m/s}, \\
\vec{u}_T &= (u_T,\ v_T,\ w_T) \text{ m/s},
\end{aligned}$$

and

$$\begin{aligned}
\rho_S &= 1.20 \text{ kg/m}^3, \\
\rho_N &= 1.15 \text{ kg/m}^3, \\
\rho_W &= 1.25 \text{ kg/m}^3, \\
\rho_E &= 1.10 \text{ kg/m}^3, \\
\rho_B &= 1.18 \text{ kg/m}^3, \\
\rho_T &= 1.05 \text{ kg/m}^3,
\end{aligned}$$

respectively.

At this instant, the density time derivative in the control volume is

$$\frac{\partial \rho}{\partial t} = -0.2 \text{ kg/(m}^3 \text{ s)}.$$

Using the integral conservation of mass equation, determine the unknown velocity component w_T.

5. Consider the three-dimensional rectangular control volume shown in Figure 5.

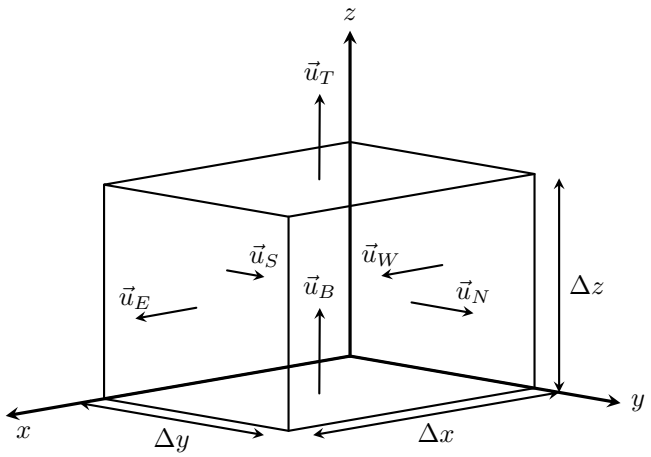


Figure 5: Three-dimensional control volume and face velocity vectors.

The control volume dimensions are

$$\Delta x = 2 \text{ m}, \qquad \Delta y = 1.5 \text{ m}, \qquad \Delta z = 1.25 \text{ m}.$$

The fluid density is uniform and equal to

$$\rho = 1.2 \text{ kg/m}^3.$$

The velocity vectors at the faces of the control volume are given by

$$\begin{aligned}
\vec{u}_S &= (0.30,\ 1.20,\ -0.15) \text{ m/s}, \\
\vec{u}_N &= (-0.20,\ 0.80,\ 0.25) \text{ m/s}, \\
\vec{u}_W &= (1.50,\ -0.30,\ 0.20) \text{ m/s}, \\
\vec{u}_E &= (u_E,\ 0.25,\ -0.10) \text{ m/s}, \\
\vec{u}_B &= (0.15,\ -0.20,\ 0.60) \text{ m/s}, \\
\vec{u}_T &= (-0.10,\ 0.35,\ 0.45) \text{ m/s}.
\end{aligned}$$

The x-component of the velocity at the east face, u_E, is unknown. At the instant considered, the rate of accumulation of x-momentum inside the control volume is

$$\frac{\partial}{\partial t} \int_{CV} \rho u \, dV = 1.35 \text{ N}.$$

The resultant pressure force acting on the control volume in the x-direction is

$$F_{p,x} = -5.500 \text{ N},$$

and the resultant viscous force in the same direction is

$$F_{v,x} = 0.304 \text{ N}.$$

Assume that body forces are negligible and that the flow is transient.

(a) Write the integral conservation of momentum equation in the x-direction.

(b) Calculate the net rate of transport of x-momentum through the control volume, expressing the result as a function of the unknown velocity component u_E.

(c) Using the momentum balance, determine the unknown velocity component u_E. Show all calculation steps.

(d) Verify whether the velocity field obtained in the previous question satisfies the integral conservation of mass equation. Calculate the net mass flow rate through the control surface and comment on the result.

6. Consider the three-dimensional rectangular control volume shown in Figure 6.

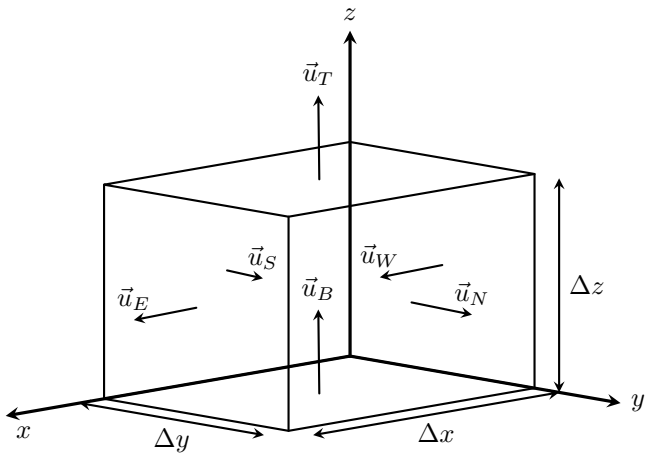


Figure 6: Three-dimensional control volume and face velocity vectors.

The control volume dimensions are

$$\Delta x = 2 \text{ m}, \qquad \Delta y = 1.5 \text{ m}, \qquad \Delta z = 1.25 \text{ m}.$$

The fluid density is uniform and equal to

$$\rho = 1.2 \text{ kg/m}^3.$$

The velocity vectors at the faces of the control volume are given by

$$\begin{aligned}
\vec{u}_S &= (0.30,\ 1.20,\ -0.15) \text{ m/s}, \\
\vec{u}_N &= (-0.20,\ 0.80,\ 0.25) \text{ m/s}, \\
\vec{u}_W &= (1.50,\ -0.30,\ 0.20) \text{ m/s}, \\
\vec{u}_E &= (-0.40,\ 0.25,\ -0.10) \text{ m/s}, \\
\vec{u}_B &= (0.15,\ -0.20,\ 0.60) \text{ m/s}, \\
\vec{u}_T &= (-0.10,\ 0.35,\ 0.45) \text{ m/s}.
\end{aligned}$$

The pressure on the bottom face is

$$p_B = 1000 \text{ Pa}$$

(gauge pressure). The pressure on the top face, p_T, is unknown.

At the instant considered, the rate of accumulation of z-momentum inside the control volume is

$$\frac{\partial}{\partial t} \int_{CV} \rho w \, dV = 1.35 \text{ N}.$$

Assume that the flow is transient and that viscous forces and body forces are negligible. The pressure is uniform over each face.

(a) Write the integral conservation of momentum equation in the z-direction.

(b) Calculate the net rate of transport of z-momentum through the six faces of the control volume.

(c) Using the momentum balance, determine the unknown pressure p_T on the top face. Show all calculation steps.