Is it really SIMPLE?

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SIMPLE

Discretized equation for the X-Component of the Momentum

$$a_P u_P = a_E u_E + a_W u_W + a_N u_N + a_S u_S + \frac{\rho_P V_P}{\Delta t} u_P^t - (p_e A_e - p_w A_w) + S_u V_P \tag{1}$$

Provisional Velocities

$$a_P u_P^* = a_E u_E^* + a_W u_W^* + a_N u_N^* + a_S u_S^* + \frac{\rho_P V_P}{\Delta t} u_P^t - (p_e^* A_e - p_w^* A_w) + S_u V_P \tag{2}$$

Corrections for Velocities

$$a_P u_P' = a_E u_E' + a_W u_W' + a_N u_N' + a_S u_S' - (p_e' A_e - p_w' A_w) \tag{3}$$

Neglecting the neighbors contributions, velocities are corrected by

$$a_P u_P' \approx -(p_e' A_e - p_w' A_w) \tag{4}$$

$$u_P' \approx \frac{p_w' A_w - p_e' A_e}{a_P} \tag{5}$$

Let us use the mass conservation equation to find pressure corrections

$$F_e - F_w + F_n - F_s = 0 \tag{6}$$

Fluxes become a provisional flux plus a flux correction

$$(F_e^* + F_e') - (F_w^* + F_w') + (F_n^* + F_n') - (F_s^* + F_s') = 0 \tag{7}$$

Simplifying!

$$(F_e^* + \rho_e A_e u_e') - (F_w^* + \rho_w A_w u_w') + (F_n^* + \rho_n A_n v_n') - (F_s^* + \rho_s A_s v_s') = 0 \tag{8}$$

Using the relationship between velocities and pressure corrections

$$\left(F_e^* + \rho_e A_e \frac{A_e(p_P' - p_E')}{a_e}\right) - \left(F_w^* + \rho_w A_w \frac{A_w(p_W' - p_P')}{a_w}\right) + \left(F_n^* + \rho_n A_n \frac{A_n(p_P' - p_N')}{a_n}\right) - \left(F_s^* + \rho_s A_s \frac{A_s(p_S' - p_P')}{a_s}\right) = 0 \tag{9}$$

Pushing provisority fluxes to the right-hand side!

$$\rho_e A_e \frac{A_e(p'_P - p'_E)}{a_e} - \rho_w A_w \frac{A_w(p'_W - p'_P)}{a_w} + \rho_n A_n \frac{A_n(p'_P - p'_N)}{a_n} - \rho_s A_s \frac{A_s(p'_S - p'_P)}{a_s} = -\left(F^*_e - F^*_w + F^*_n - F^*_s\right) \tag{10}$$

Simplifying by introducing the d_f coefficients

$$d_e(p'_P - p'_E) - d_w(p'_W - p'_P) + d_n(p'_P - p'_N) - d_s(p'_S - p'_P) = -\left(F^*_e - F^*_w + F^*_n - F^*_s\right) \tag{11}$$

Simplifying by distribution

$$d_e p'_P - d_e p'_E - d_w p'_W + d_w p'_P + d_n p'_P - d_n p'_N - d_s p'_S + d_s p'_P = -\left(F^*_e - F^*_w + F^*_n - F^*_s\right) \tag{12}$$

Organize terms by variable

$$\left[d_e + d_w + d_n + d_s\right] p'_P - \left[d_e\right] p'_E - \left[d_w\right] p'_W - \left[d_n\right] p'_N - \left[d_s\right] p'_S = -\left(F^*_e - F^*_w + F^*_n - F^*_s\right) \tag{13}$$

Congratulations, you have a pressure correction equation!

$$\boxed{a^p_P p'_P - a^p_E p'_E - a^p_W p'_W - a^p_N p'_N - a^p_S p'_S = b_p} \tag{14}$$